Given a graph of f, how do you sketch a graph of f' ?

When first learning to sketch a graph of f' given the graph of f,
if you try to do it completely intuitively, it is common to accidentally sketch the graph of f shifted up or down instead.

The following process breaks down the steps of sketching f' in a more structured way.

[1] Identify all $x-$coordinates on the graph of f where there is
- a horizontal tangent line
  - so $f' = 0$
- a discontinuity, a cusp or a vertical tangent line
  - so f' does not exist

[2] Identify all $x-$coordinates on the graph of f where
- the graph is the steepest or flattest in that neighborhood

[3] On a number line, mark all $x-$values from [1] and [2]

At each $x-$value from [1] where
- f has a horizontal tangent line
  - draw a dot on the number line
- f has a vertical tangent line or infinite discontinuity
  - draw a vertical asymptote
- f has a discontinuity or cusp
  - indicate that there is no corresponding point on the graph of f'

[4] For each subinterval of the number line in [3],
label whether
- f is increasing
  - so $f' > 0$
- f is decreasing
  - so $f' < 0$
- f is horizontal
  - so $f' = 0$

Also, label whether
- f is getting steeper
  - so f' is getting larger in size
- f is getting flatter
  - so f' is getting smaller in size
- f is straight
  - so f' is not changing

[5] For each subinterval of the number line in [3],
sketch a piece of the graph of f' such that

if $f' > 0$, the graph of f' is __________ the number line

if $f' < 0$, the graph of f' is __________ the number line

if $f' = 0$, the graph of f' is __________ the number line

if f' does not exist, the graph of f' has a ______________________________
if f has a vertical tangent line or infinite discontinuity

the graph of f' has a __________
if f has a discontinuity or a cusp
without a vertical tangent line or infinite discontinuity

if f' is large in size, the graph of f' is ____________________ the number line

if f' is small in size, the graph of f' is ____________________ the number line

if f' is getting larger in size, the graph of f' is moving ____________________ the number line

if f' is getting smaller in size, the graph of f' is moving ____________________ the number line

if f' is not changing, the graph of f' is ____________________

[6] At each $x-$value in [3] where f' exists
join up the pieces of f' on the left and right sides of that $x-$value
paying attention if $f' = 0$ at that $x-$value

At each $x-$value in [3] where f' does not exist due to a jump or removable discontinuity
if the graph of f has the same slope as it approaches that $x-$value from the left and from the right
join up the pieces of f' on the left and right sides of that $x-$value to meet at __________
if the graph of f has different slopes as it approaches that $x-$value from the left and from the right
pay attention to which side of f is steeper